CAMILLE BAKER

How Big Was the Roman Empire?

THIS ACTIVITY WAS DESIGNED AS PART OF A sixth-grade interdisciplinary unit, "Seeing the World through the Eyes of Ancient Greeks and Romans." In addition to learning about Greek and Roman geography, economics, government, and societies in social-studies class, students studied ancient scientists, physicians, and inventors in science class. They also explored Greek and Roman myths, religions, languages, and ideas in language-arts classes. In mathematics classes, students experimented with the golden ratio and the pentagram, wrote an essay on how the Greeks used mathematics to understand their world, examined Greek and Roman architecture, and investigated the physical size of the Roman Empire. To culminate the unit, students worked in small groups on special projects, such as building a scale model of the Parthenon, measuring and creating a scale drawing comparing the soccer field with the Pantheon, creating and performing original myths or plays depicting life in ancient Greece and Rome, and constructing simple machines or demonstrations of the scientists' work in Greek and Roman times.

Students wrote about the mathematical connections and inconsistencies they discovered

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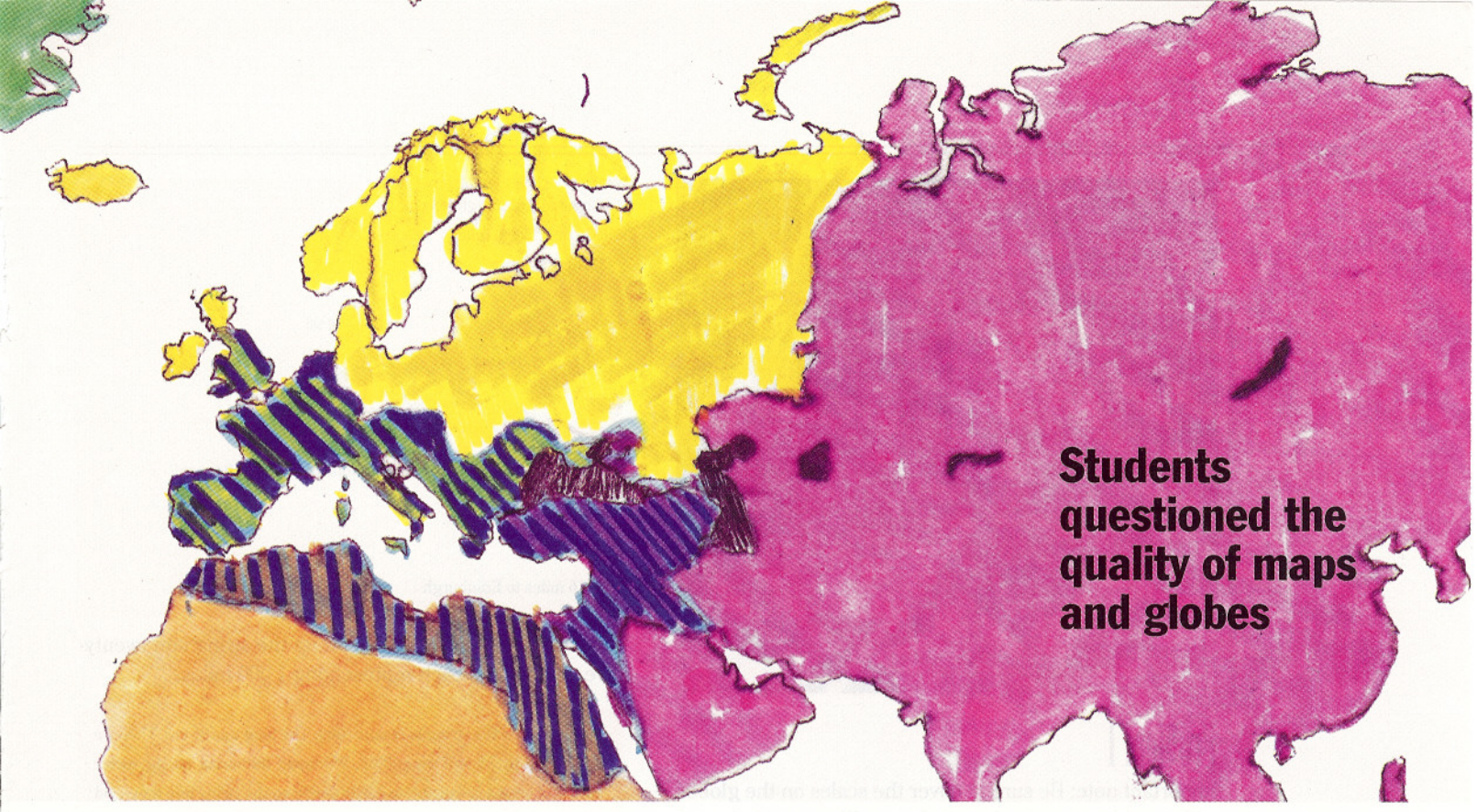
In this cooperative, problem-solving activity, students share ideas, choose strategies, reach solutions, and draw conclusions using a variety of mathematical approaches. When students tackle complex problems in a geographical, historical, or real-world context, they live the problems instead of just thinking of them as tasks to finish or drills to master. The activity's instructions are intentionally sparse to allow student groups to direct their own unique, problem-solving processes and share their work with the entire class.

The Students' Work

GROUPS OF THREE TO FIVE STUDENTS WORKED ON each question in **figure 1.** They put their heads together not only to find solutions but also to write down their collective thinking processes. They had prepared for writing such group papers by keeping individual mathematics logs of everyday activities. Because the teacher had previously assessed the clarity and effectiveness of their log entries, they knew what was expected of them in the group presentation.

Before beginning the Roman Empire activity, students had worked with fractions, percents, area, and scale. However, they had worked with ratios only in the form of equivalent fractions. As the groups shared their results with the entire class, students were astonished at the connections among division, fractions, map scales, percents, and ratios.

They also found inconsistencies in their solutions; this dilemma motivated students to check one another's measurements, strategies, and calculations. When they were



unable to resolve some of the inconsistencies, the students questioned the quality and accuracy of maps, globes, and measurement tools. The teacher and students discussed how maps and globes are tiny models of a very large world; measurement tools are inaccurate on such models. They concluded that maps and globes are probably not accurately scaled-down versions of the land and water areas of the world.

After the group sharing, students had the opportunity to write in their individual mathematics logs about the mathematical connections and inconsistent solutions they discovered as groups and as a class. They also wrote about how they would proceed differently if they were to redo the activity. Both the group paper and the individual log entries helped the teacher assess each student. Samples of the papers describing the group processes follow. (Note: students' spelling and punctuation have been retained throughout this article.)

How Big Was the Roman Empire?

I. Using a piece of string, a globe, and the knowledge that the circumference of the earth is about 25 000 miles, find the distance from Rome to the following outposts of the Roman Empire. The present-day names of the outposts are given.

Northern outpost:	Edinburgh, Scotland
Southern outpost:	Aswan, Egypt
Eastern outpost:	Yazd, Iran
Western outpost:	Casablanca, Morocco

II. Use a copy of the student atlas to estimate the following areas (hint: use tracing paper).

1. About what percent of present-day Europe did the Roman Empire cover?
2. About what percent of present-day Africa did the Roman Empire cover?
3. About what percent of present-day Asia did the Roman Empire cover?
4. About what percent of the world's land mass did the Roman Empire cover?

III. Estimate the area of the Roman Empire

IV. The size of the Roman army is said to have been about 300 000 men. How can the relationship between the size of the empire and the size of the army be described? Considering this relationship, what conclusions might be drawn about the fate of the Roman Empire?

V. Extra for experts! Determine the solutions in kilometers (or in miles if kilometers were used first).

Fig. 1 Activity questions

Problem 1

Important note: Be sure to cover the scales on the globes with masking tape so that students will create their own scales.

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Northern outpost:	Edinburgh, Scotland
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Group 1 used division to create a scale.

I. In order to figure out this problem we had to do some caculations. We figured out that the earth is 25,000 miles. We took a piece of string and wrapped it around the globe, finding out that on the globe that the earth was 94 cm. by measuring the string.

A) Taking the string we found out that Ediburgh, Scotland is 5 cm. away from Rome on the Globe. We now have to figure out how many 5 cm equals in miles. How many miles do you think that would be in real life. Using the information we talked about before, we know that 94 cm = 25,000 miles. So on the calculator we divided 25,000 by 94. We got 265.95 which we rounded to 266. This means that each cm on the globe equals 266 miles. In knowing this we multiplied 266 × 5 which equals 1,330. So Rome is about 1,330 miles from Edinburgh, Scotland.

Group 2 used fractions. Each outpost distance was a fractional part of the string and thus a fractional part of 25 000 miles.

Rome to Edinburgh = 5 cm.

95 cm = 25,000 miles

5 cm = 19 times

$$\frac{1}{19} \text{ of } 95$$

$$\frac{1}{19} \text{ of total}$$

The distance to Edinburgh from Rome fits into it 19 times.

25,000 ÷ 19 = 1315.7

1316 miles to Edinburgh

Group 3 created a scale by cutting the string into twenty-five equal parts, each representing 1 000 miles.

We measured the string in centimeters. The total was 95 cm. Then we divided 95 by 25 and we got 3.8. We taped the string on the meter stick and used a calculator, to easily divide up the string into thousand mile differences. Each space is 3.8 cm., so each 3.8 section of string is 1,000 miles. Rome to Edinburgh was about 1,300 miles.

Group 4 used ratios.

We measured the globe around. It was 50 inches. We then measured Rome to Edinburgh. It was 2.5 inches. The toatal miles in the circumference of the world is 25,000 miles. We knew that 2.5 inches had the same relationship to 50 inches, as X had to 25,000 miles. We wrote:

$$\frac{2.5}{50} = \frac{X}{25{,}000}$$

We multiplied 2.5 by 25,000 and divided by 50. We got 1,200 miles from Rome to Edinburgh.

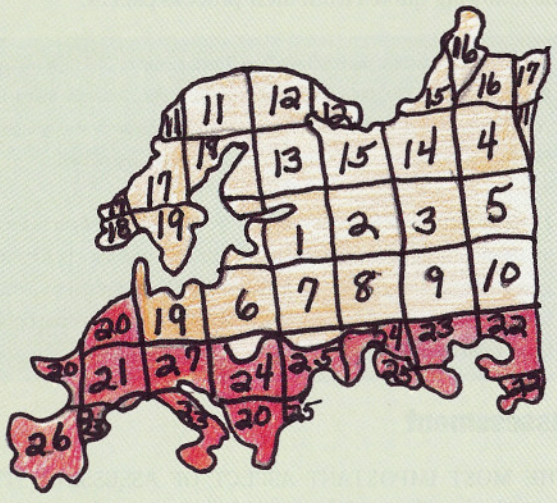
Problem 2

Important note: Some students followed the hint and traced the outlines of the continents on tracing paper placed over atlas maps. They then taped the tracings over centimeter-grid paper to estimate the areas.

II. Use a copy of the student atlas to estimate the following areas (hint: use tracing paper):

1. About what percent of present-day Europe did the Roman Empire cover?
2. About what percent of present-day Africa did the Roman Empire cover?
3. About what percent of present-day Asia did the Roman Empire cover?
4. About what percent of the entire world's land mass did the Roman Empire cover?

Group 1 made a fraction of the square units of area of the Roman Empire compared with the continent areas and then used ratios to arrive at percents of the continents covered by the empire. Their work comparing the European part of the Roman Empire with Europe follows.



What we did was trace an approximate perimeter at Europe. We shaded it in yellow. Then on top of Europe, we traced the Roman Empire and shaded it in red. Then we counted up the total amount of squares in Europe which was 27 and the Roman empire was in 8 of those squares. Our equasion was

$$\frac{8}{27}.$$

We had to change it into a percent so we knew that $\frac{\quad}{100}$ = % so now our equasion looked like this:

$$\frac{8}{27}=\frac{\quad}{100.}$$

We could do equivelent fractions. Then the equasion wasld look like this:

$$\frac{8}{27}=\frac{30}{100}\text{ (estimated)}$$

We can cross multiply to check our answer which shows that the two equasions is

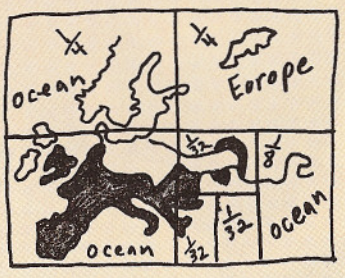
$$\frac{8}{27}=\frac{30}{101.25}=30\%$$

Group 2 cut the land areas with which they were comparing the Roman Empire into increasingly smaller fractional parts, added the parts for the Roman Empire, and then used ratios to change the fractions to percents. Their work for Europe is shown.

II. We had a different strategy than most people. Instead of using graph paper we drew a part of land and then cut into 1/2's, 1/4's, 1/8's, etc. until the part of land exactly filled in a box. Sometimes we had a little room left over for we included water, so we took some other parts of land and pretended that they fit together so the box was complete so it would be more accurate.

$$\frac{1}{4}+\frac{1}{32}+\frac{1}{32}=\frac{10}{32}=\frac{5}{16}$$

So it's about 30% of Europe. That was our strategy and we hope you like it!

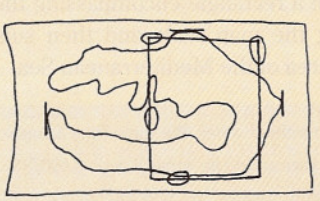


Group 3 estimated the parts of the Roman Empire compared with the continents simply by seeing how many index fingers were required to cover the empire compared with how many index fingers were required to cover each continent. (See photograph on page 758.)

We took the map of Europe and took our fingers and layed them down on Europe to see how many fingers equaled the area of Europe. It ended up to be 4 fingers. Then we did the same thing to the Roman Empire and came out to one figger. So from that we can tell that the ancient Roman empire equle one forth of Europe today which equals 25%. Then we did it for Asia and we got 10 finger lengths. but for the ancient Roman Empire it equaled half a finger length which equals 1/20 of Asia..

Group 4 used estimated rectangle areas. They imagined pushing together the land areas into rectangles and replacing the water and then figured the area of the rectangles. Next they compared the rectangular areas for the Roman Empire with the rectangular areas for the continents.

To find the percentage of the world the Roman Empire covers, we have 2 theories to find the answer. Our theory is to make the Roman Empire into a square and take away the extra water.



The cirles are the middle of the sides. This is a rough drawing of the Roman Empire squared. Our other theory is to copy the Empire on transparent graph paper and then find the extremes, (the longest and tallest point), and make a rectangle out of it. Then we–the extra area from the rectangle. That made the Roman Empire squared. Then we compared the squared to the fourth of the world, which equaled

$$\frac{1}{20}\times 4=\frac{1}{80}\text{ of the world or }1\frac{1}{4}\%!$$

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Problem 3

III. Estimate the area of the Roman Empire

Groups 1, 2, and 4 used the map-scale value for one centimeter, calculated the number of square miles in a square centimeter, and multiplied that value by the number of centimeter squares covered by the Roman Empire.

> This is how we got square miles:
> We found the scale and 1 cm equals 650 miles
> We made a box and it equals 650 × 650 or 422,500 sq. mi. We estimated it to 420,000. Then since there were about 10 squares filling the Roman Empire we multiplied 10 by 420,000 and got the answer which is 4,200,000 sq. mi. in the Roman Empire
>
> □ 1 cm = 650 mi.
> 1 sq. cm = 422,500 sq. mi.
>
> Roman Empire = 10 sq cm.
> Roman Empire = 4,200,000 sq. mi.

Group 3 found the area of a rectangle encompassing the Roman Empire by using the map scale and then subtracted the approximate area of the Mediterranean Sea.

> 8,000,000 with water, 6,400,000 without water. We turned the Roman Empire into a rectangle and measured the area, it was 8,000,000 miles. Then we turned the Mediterranean Sea into a rectangle and measured its area. It was 1,600,000. I subtracted and got 6,400,000.

Problem 4

IV. The size of the Roman army is said to have been about 300 000 men. How can the relationship between the size of the empire and the size of the army be described? Considering this relationship, what conclusions might be drawn about the fate of the Roman Empire?

Note that even though different groups used different figures taken from various calculations in problems 1, 2, and 3, students used the same basic approach to this problem and arrived at the same general conclusion, as seen in the following quotes from their process papers.

> We divided the total square miles of the Roman Empire (4.5 million) by the Roman Army (300,000) and you get 1 soldier per 15 square miles. If you compare the size of the empire by the size of the army you will see that there are too few soldiers to cover such a large area of land.
>
> I realized math has a lot to do with history. With math we figured that its [Rome's] biggest enemy was itself. The land was too big for the army. If any invading people came there might be a easy defeat. With math, I realized, anything is possible to figure out if you have the right figures and methods.

Assessment

THE MOST IMPORTANT ASPECT OF ASSESSING STUdents is that they think about their own performance—what they have learned and how they can improve—instead of about what grade they receive. All too often, grades encourage students to focus on the rating instead of the learning. If students are involved in a process of self- and peer-assessment before the grades are assigned, they get the message that learning and improvement are valued. An articulate self-analysis or critique that reveals valuable learning from experience and that is reflected in the student's mathematics-log entry might even be included in the formulation of a "grade."

The students were rated on a scale of 1 through 6 for their performance on the following criteria. Their performance could also have been described by using such adjectives as *always, frequently, sometimes,* and *not yet* for each criterion.

Effective collaboration

- Did a good balance of listening and talking by every member of the group occur?
- Did the group check along the way to ensure the understanding and agreement of all members of the group?
- If the group agreed to subdivide, how well did they communicate and combine their thinking?

Mathematics

- How effective were the strategies? How well did the students adjust strategies or adopt new ones when the

initial strategies proved ineffective?
- Was the mathematical reasoning logical?
- Were the solutions reasonable and relatively accurate?

Written communication

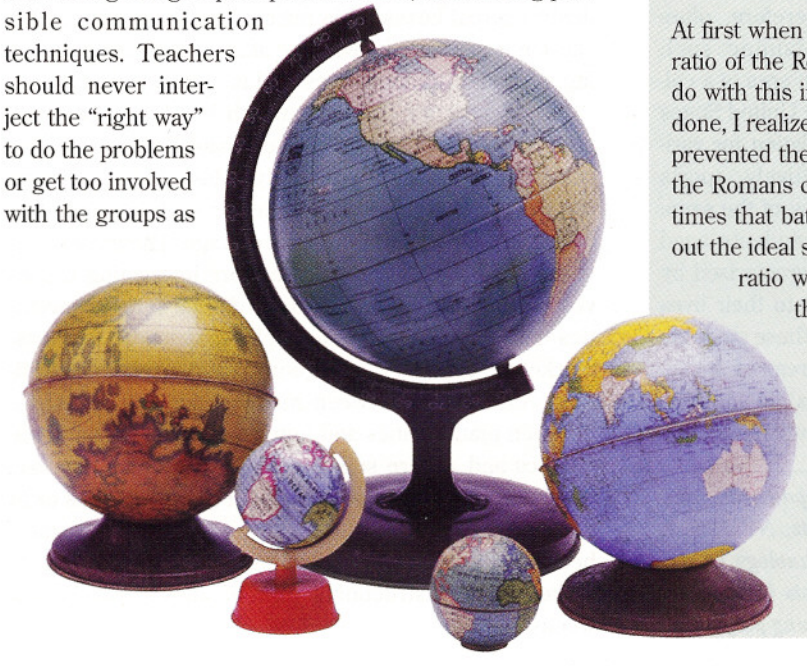
- Was the process through which the students arrived at their solutions communicated clearly and thoroughly?
- Were all computations and calculator work included in the written process?
- Was it easy to connect the map work to the written process?

Oral communication (sharing with the entire class)

- Did everyone in the group contribute to the sharing process?
- Was the presentation of information easily understood?
- Were the presentation materials, such as the overhead projector, chalkboard, manipulatives, large paper, and demonstrations, effective?
- Did the students speak so that they could be heard well? Did they make eye contact with members of the class?

The Teacher's Role

WHILE STUDENTS (1) CLARIFY THE PROBLEMS, (2) DEvelop and implement courses of action, (3) arrive at solutions, and (4) plan how to communicate their processes and conclusions, the teacher functions as an advisor, asking guiding questions about the mathematics, making suggestions about such cooperative techniques as paraphrasing and questioning, observing and assessing problem-solving and group-cooperation skills, and offering possible communication techniques. Teachers should never interject the "right way" to do the problems or get too involved with the groups as they grapple with ideas and strategies. Students learn from mistakes and by backtracking on ineffective strategies. When the student groups share mistakes, detours, and different paths to success in problem solving, they learn the value of risk taking and become more confident and varied in their thinking.



Conclusion

THE BEST CONCLUSION FOR "HOW BIG WAS THE Roman Empire?" is the students' own summations in their mathematics logs. Both their questions and reflections are an excellent gauge of students' thinking and catalysts for further exploration.

How do you make a flat map of a curved space?

Does the area stay the same if the perimeter stays the same but the shape is different?

I learned that there's more than one way to get the area of something, and that you can get the area of just about anything you want!

I learned that ratios, fractions, decimals, and percentages all connect and can measure a certain part of an object. I now know everything in math is useful all around the world and is used every second in life.

This project taught us to use things other than scales to measure distance. And invent totally new strategies in the process. We also got our group to get along with each other.

At first when I began to figure out the soldier/sq. mile ratio of the Roman Empire, I wondered what we could do with this information to make it useful. When I was done, I realized that this information could actually have prevented the Roman Empire from being conquered. If the Romans compared the soldier/sq. mile ratio to the times that battles were won and lost, they could figure out the ideal size for the army. For example, if when the ratio was 1/30 the Romans lost all their battles, then they would know to increase the size of the army. On the other hand, if when the ratio was 1/10 the Romans won every battle, they could send some men to become farmers and craft workers. (Note: This student placed this entry in his portfolio as an example of his best work because "I feel I was able to make the connection for what the man/mile ratio could be used for.") ▲